A proof of the logical equivalence of inverse and converse

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The inverse of p → q is ¬p → ¬q, and its converse is q → p.

This logical proof shows that the converse of a conditional entails its inverse (i.e., that q → p ⊢ ¬p → ¬q):

```
[1] q → p      // premise
[2] ¬q ∨ p     // disjunction from conditional (1)
[3] p ∨ ¬q     // commutativity of disjunction (2)
[4] ¬¬p ∨ ¬q   // double negation (3)
[5] ¬p → ¬q    // conditional from disjunction (4)
```

Its reversal proves that inverse entails converse (¬p → ¬q ⊢ q → p):

```
[1] ¬p → ¬q    // premise
[2] ¬¬p ∨ ¬q   // disjunction from conditional (1)
[3] p ∨ ¬q     // double negation (2)
[4] ¬q ∨ p     // commutativity of disjunction (3)
[5] q → p      // conditional from disjunction (4)
```

Because inverse and converse entail each other (q → p ⊣⊢ ¬p → ¬q), they are logically equivalent: q → p ≡ ¬p → ¬q